The midterm exam will cover chapters 1, 2, 3, and 4 of our text, and will be an in-class exam on Friday, October 14. For the exam you should:

be able to define:

- integral domain;
- field;
- Euclidean domain;
- principal ideal domain (PID);
- unique factorization domain (UFD);
- Noetherian ring;
- unit in a ring;
- zero divisor in a ring;
- nilpotent element of a ring;
- irreducible element in an integral domain;
- prime element in an integral domain;
- comaximal ideals;
- radical of an ideal;
- nilradical of R;
- ideal quotient (colon ideal);
- maximal ideal;
- prime ideal;
- primary ideal;
- decomposable ideal;
- irreducible ideal;
- minimal prime ideal of I;
- embedded prime ideal of I;
- minimal primary decomposition;
- $\mathrm{Spec}(R)$;
- $\mathrm{Var}(I)$;
- $\mathrm{Min}(I)$;
- $\mathrm{ass}_R(I)$;

be able to state:

- Chinese remainder theorem (see the online lecture notes for the statement);
- First uniqueness theorem for primary decomposition;
- Second uniqueness theorem for primary decomposition;

understand and know:

- The natural map surjective map $R \to R/I$ that has kernel I;
- The isomorphism theorem;
- Given an ideal I of R, the bijection between the ideals J such that $I \subseteq J$ and the ideals of R/I;
- Given a ring homomorphism $f : R \to S$, the contraction and extension maps on ideals determined by f, especially when f is surjective;
- How to characterize an ideal I of R being maximal, prime, or primary in terms of the quotient R/I;
- The equivalent conditions for an ideal (r) in a PID to be a prime ideal;
- If R is an Euclidean domain, then R is a PID;
- If R is a PID, then R is a UFD;
- If K is a field, $K[X]$ is Euclidean, hence a PID, hence a UFD;
- If R is a UFD, then $R[X]$ is a UFD;
- Every proper ideal is included in a maximal ideal;
- $\sqrt{I} = \bigcap_{P \in \mathrm{Var}(I)} P = \bigcap_{P \in \mathrm{Min}(I)} P$;
- Formulas involving radicals:
  $\sqrt{IJ} = \sqrt{(I \cap J)} = \sqrt{I} \cap \sqrt{J}$;
  $\sqrt{P^n} = P$ if P is prime;
- $\sqrt{Q}$ maximal $\implies$ Q primary;
- M maximal $\implies$ M^n primary;
- What are the primary ideals in a PID;
- If R is Noetherian, then every proper ideal has a primary decomposition;

and be able to demonstrate knowledge of the above concepts and results in the context of simple problems and questions for the examples $\mathbb{Z}$, $\mathbb{Z}/N\mathbb{Z}$, $K[X]$, and $K[X_1, \ldots, X_n]$.